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DISSOLUTION OF A SOLID PHASE BY FLUID FLOWING IN A CYLINDRICAL PIPELINE

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The problem of mass transfer from the inner surface of a cylindrical pipeline in the presence of dissolution of a solid phase in the turbulent fluid flow is examined.

The heterogeneous transformation surface relative to the inner wall of a pipeline is

$$h = h_0(1 - \varepsilon_0 \cos\varphi)(1 + k\eta). \tag{1}$$

The distribution (1) occurs in main pipelines after they are cleaned with heavy mechanical devices – scrapers and separators. Due to their intrinsic weight, solid deposits are more completely removed from the bottom of the pipe: $\varphi = 0$, $h = h_0(1 - \varepsilon_0)(1 + k\eta)$ is the smallest thickness of the deposits along the lower generatrix of the pipe; $\varphi = \pi$, $h = h_0(1 + \varepsilon_0)(1 + k\eta)$ is the greatest thickness along the upper generatrix, $0 \le \varphi \le \pi$. The factor $(1 + k\eta)$ takes into account the change in thickness of the solid phase along the pipe as a result of deformation and wear of the packing elements of the separators and scrapers.

Particular cases of the problem proposed are examined in [1], which is concerned with the problems of mass transfer in main pipelines.

We are examining the case of large diffusion Prandtl numbers $\mathrm{Pr} = \nu/D$. Then, the concentration of the impurity in the fluid will change within the viscous sublayer [2] and for a one-dimensional stabilized flow, its average (over the cross section of the pipe) value can be determined from the following equation [3-5]:

$$\frac{\partial\Theta_i}{\partial\tau} + \frac{\mathrm{Pe}}{2}\frac{\partial\Theta_i}{\partial\eta} - \mathrm{St}\,\mathrm{Pe}(\Theta_\omega - \Theta_i) = 0, \quad i = 1 \ \text{for} \ \frac{\mathrm{Pe}}{2}(\tau - \tau_1) \leqslant \eta \leqslant \frac{\mathrm{Pe}}{2}\tau, \quad i = 2 \ \text{for} \ \xi_1(\tau) \leqslant \eta < \frac{\mathrm{Pe}}{2}(\tau - \tau_1). \tag{2}$$

With the appearance of a clean pipe surface, for any cross section, Eq. (2) has the form

$$\frac{\partial\Theta_3}{\partial\tau} + \frac{\mathrm{Pe}}{2}\frac{\partial\Theta_3}{\partial\eta} - \mathrm{St}\,\mathrm{Pe}(\Theta_\omega - \Theta_3)\left(1 - \frac{\varphi}{\pi}\right) = 0, \tag{3}$$

which stems from the form of the distribution of the third phase (characteristic (1)) and occurs for $\xi_1(\tau) \le \eta \le \xi_2(\tau)$. Here and in (2) above, $\xi_1(\tau)$ and $\xi_2(\tau)$, which are functions determined from the conditions $\varphi(\xi_1(\tau), \tau) = 0$, and $\varphi(\xi_2(\tau), \tau) = \pi$, indicate the boundary coordinates of the clean border of the pipe. The impurity concentration function is continuous along the pipe, so that the following boundary conditions are valid:

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$$\Theta_1 = \Theta_2 \quad \text{for } \eta = \frac{\text{Pe}}{2}(\tau - \tau_1); \quad \Theta_2(\xi_1(\tau), \tau) = \Theta_3(\xi_1(\tau), \tau);$$
$$\Theta_3(\xi_2(\tau), \tau) = 0. \tag{4}$$

In solving (1)-(4), we used the equation of the kinetics of dissolution [6]

$$\frac{dH}{d\tau} = \frac{\text{Nu}}{4}(\Theta_\omega - \Theta). \tag{5}$$

We are examining the case $h \ll 1$. Then, the reaction surface (Eq. (1)) can be represented as follows: $h = h_1(1 + 2\varepsilon_1\varphi/\pi)(1 + k\eta)$, where $h_1 = h_0(1 - \varepsilon_0)$, $\varepsilon_1 = \varepsilon_0/(1 - \varepsilon_0)$.

The solution of (1)-(5) reduces to finding the solution of a differential equation for the function $\varphi(\eta, \tau)$, characterizing the geometry of the moving separation boundary:

$$\frac{\partial\varphi}{\partial\tau} + \frac{\text{Pe}}{4}\frac{\partial\varphi}{\partial\eta} + \left(\text{Nu} + \frac{\text{Pe}}{4}\frac{k}{1 + k\eta}\right)\varphi - \frac{\text{Nu}}{2\pi}\varphi^2 = -\frac{\pi}{2\varepsilon_1}\left(\text{Nu} + \frac{\text{Pe}}{4}\frac{k}{1 + k\eta}\right). \tag{6}$$

The solution of (6) has the following form:

$$\frac{\varphi}{\pi} = \frac{\exp\left\{\int\left(4\text{St} + \frac{k}{1 + k\eta} - \frac{4}{\pi}\text{St}\, y(\eta)\right)d\eta\right\}}{\pi\psi\left(\frac{\text{Pe}}{4}\tau - \eta\right) + 2\text{St}\int\exp\left\{\int\left(4\text{St} + \frac{k}{1 + k\eta} - \frac{4}{\pi}\text{St}\, y(\eta)\right)d\eta\right\}d\eta} + y(\eta),$$

and $y(\eta)$ satisfies Eq. (7)

$$\frac{\text{Pe}}{4}\frac{dy}{d\eta} + \left(\text{Nu} + \frac{\text{Pe}}{4}\frac{k}{1 + k\eta}\right)y - \frac{\text{Nu}}{2\pi}y^2 = -\frac{\pi}{2\varepsilon_1}\left(\text{Nu} + \frac{\text{Pe}}{4}\frac{k}{1 + k\eta}\right). \tag{7}$$

The function $\psi(\text{Re}/4(\tau - \eta))$ is determined from the condition $\varphi(\xi_1(\tau), \tau) = 0$.

The numerical solution of (7) shows that for $k \sim 10^{-4} - 10^{-5}$ the function y can be assumed to be a constant equal to $\pi(1 + \varepsilon_0^{-0.5})$. In this case

$$\frac{\varphi}{\pi} = 1 + \varepsilon_0^{-0.5} - \frac{2\varepsilon_0^{-0.5}(1 + k\eta)}{1 + k\eta + \frac{k\varepsilon_0^{-0.5}}{4\text{St}} - \Phi^{\varepsilon_0^{-0.5}}\left\{\frac{k\varepsilon_0^{0.5}}{4\text{St}} - \frac{1}{\alpha}\left(1 - \frac{k}{4\text{St}}\ln\Phi\right)\right\}\exp\left\{\frac{4\text{St}}{\varepsilon_0^{0.5}}\eta\right\}}.$$

Here

$$\Phi = 1 + \frac{2\varepsilon_0^{0.5}}{(1 - \varepsilon_0)\ln\alpha}\left[1 - \exp\left\{\frac{(1 - \varepsilon_0)\ln\alpha}{2\varepsilon_0^{0.5}}\left(1 - \frac{\Theta_\omega}{h_0}\text{St}\left(\frac{\text{Pe}}{4}\tau - \eta\right)\right)\right\}\right]; \quad \alpha = \frac{1 + \varepsilon_0^{0.5}}{1 - \varepsilon_0^{0.5}}.$$

The expression found permits determining, for given dimensions of the reaction surface, from the condition $\varphi = \pi$ the duration of the process of dissolution of the solid phase from the inner surface of pipeline. For $k = 0$, it goes over into the equation for the dependence of the dissolution time of the solid phase, distributed relative to the inner wall of the pipe according to the law $h = h_0(1 - \varepsilon_0 \cos\varphi)$ (Eq. (1)); if at the same time $\varepsilon_0 \to 0$, we obtain the dissolution time $\tau = 4\left\{\frac{1}{\text{Nu}}\frac{h_0}{\Theta_\omega} + \frac{1}{\text{Pe}}\left(1 + 4\frac{h_0}{\Theta_\omega}\right)\right\}$ for $h = h_0$ [1].

NOTATION

Here $h = \delta/d$, $h_0 = \delta_0/d$, $\eta = x/d$; δ_0, average (over the cross section of the pipe) thickness of the solid substance; $d = 2R$, diameter of the pipeline; x, longitudinal coordinate; φ, polar angle; ε_0, constant ($0 < \varepsilon_0 \leq 1$); ν, kinematic coefficient of molecular viscosity; D, coefficient of molecular diffusion of the impurity; Θ_i, average concentration of the impurity in the core of the flow; Θ_ω, concentration of the impurity on the surface of dissolution, which is constant within the diffusion sublayer; W, average velocity of the flow; j_ω, impurity flux from the dissolution surface; $\tau = tD/R^2$, dimensionless time; $\tau = t_1D/R^2$, time at which the border of the clean pipe surface begins to form at the initial section as a result of total transformation of the solid phase into the solvent flow; H, thickness of the solid phase layer; k_p, rate coefficient for dissolution; $k_p = D/\Delta$, Δ, thickness of the diffusion sublayer; $\text{Nu} = k_pd/D$, diffusion Nusselt number; $\text{Re} = \bar{w}d/\nu$, Reynolds number; Pe = RePr, diffusion Peclet number; $\text{St} = j_\omega/\rho\bar{w}(\Theta_\omega - \Theta)$, Stanton number.

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MASS TRANSFER FROM A MOVING BUBBLE DURING A SLOW CHEMICAL REACTION

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A previously proposed method for solving inhomogeneous problems in the theory of heat and mass transfer is refined. As an illustration, the stationary mass transfer from a moving bubble during a slow chemical reaction of first or second order is examined.

We shall examine the problem

$$\frac{\partial C}{\partial \tau} - \frac{\partial^2 C}{\partial \xi^2} = Q(\xi,\ \tau), \quad 0 \leqslant \xi < \infty,\ 0 < \tau < \infty, \tag{1}$$

$$C|_{\xi=0} = C_s(\tau); \quad C|_{\xi=\infty} = 0; \quad C|_{\tau=0} = 0, \tag{2}$$

which describes mass transfer in a semiinfinite region under the action of a source. It is necessary to find the quantity $q_s = (\partial C/\partial \xi)_{\xi=0}$, which determines the mass flux through the boundary of the region.

As in [1], we shall represent Eq. (1) in the form

$$\left(D^{1/2} - \frac{\partial}{\partial \xi}\right)\left(D^{1/2} + \frac{\partial}{\partial \xi}\right) C = Q(\xi,\ \tau), \tag{3}$$

where the fractional differentiation operators are defined by the expressions

$$D^{\nu} f(\tau) = \frac{1}{\Gamma(1-\nu)} \frac{d}{d\tau} \int\limits_0^{\tau} (\tau - z)^{-\nu} f(z)\, dz, \quad -\infty < \nu < 1.$$

The concentration gradient sought at the boundary is obtained as follows [1]. We apply the operator inverse to $D - \partial/\partial\xi$ on the left side of Eq. (3). For $(D - \partial/\partial\xi)^{-1}$, we previously found an expression in the form of an infinite series. It turns out that the inverse operator can also be written in the form

$$\left(D^{1/2} - \frac{\partial}{\partial \xi}\right)^{-1} f(\xi,\ \tau) = \int\limits_{\xi}^{\infty} e^{-(\eta-\xi)D^{1/2}} f(\eta,\ \tau)\, d\eta. \tag{4}$$

The following expression, defined in [2], enters into the operator in the integrand:

$$e^{-aD^{1/2}} f(\xi,\ \tau) = \frac{d}{d\tau} \int\limits_0^{\tau} \operatorname{erfc}\left(\frac{a}{2\sqrt{\tau - z}}\right) f(\xi,\ z)\, dz. \tag{5}$$

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